$$\sigma_c = \sigma_{c0}\left(1 - \frac{RT}{U}\ln\frac{\tau}{\tau_0}\right),$$

where τ is the time from the moment that the bottom hole passes the volume of rock examined; σ_{c_0} and τ_0, some experimentally determined constants, which do not depend on temperature and time; T, absolute temperature; R, gas constant; U, activation energy. In this case, the loss of stability and cavern formation will begin in the higher sections of the unreinforced well in the homogeneous rock; layered rocks can be taken into account using the same equations.

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PERIODIC PROBLEM OF THE INTERACTION OF SYSTEMS OF CIRCULAR OPENINGS AND STRINGERS

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Problems involving interaction of different types of concentrators, viz., openings, cuts, rigid edges (stringers), arising in technology have been the subject of a number of investigations, which are reviewed in [1, 2]. In particular, the interaction of an opening with one and two stringers was examined in [1, 3], and the interaction of a periodic system of cuts and stringers was examined in [4].

In this paper, we examine the mutual effect of a periodic system of circular openings, situated along a straight line, and a periodic system of stringers, orthogonal to this straight line. In this case, it is important to combine the methods in [1, 4, 5], developed for singular concentrators, with the techniques for solving problems on the weakening of a surface by an opening and a periodic system of openings [6, 7].

We shall examine a plate, consisting of a periodic system of circular openings and a periodic system of stringers (Fig. 1). The centers of the openings γ_k ($k = 0, \pm1, \pm2, \ldots$) are situated on the straight line $y = 0$ at the points $x_k = 2kb$, and the radii of the openings equal ρ ($\rho < b$). The stringers Γ_k continuously fixed to the plate have the same length $2a$ ($a < b$), perpendicular to the straight line $y = 0$ and intersected at the points $x_k = (2k + 1)b$. The stringers do not resist bending and function only under tension; E, ν, and h are, respectively, the elastic modulus, Poisson's coefficient, and the thickness of the plate; E_0, and S_0 are the elastic modulus and surface area of the transverse section of a stringer.

For the elements in the elastic fields the following notation is used: σ_x, σ_y, τ_{xy}, stress components; u, v, components of the displacements of the plate; N(y), normal force in the section of a stringer; $\varepsilon^0(y)$, relative elongation of its axis.

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The following stretching forces are applied to the plate

$$\sigma_y^\infty = p = \text{const}, \quad \sigma_x^\infty = \tau_{xy}^\infty = 0. \tag{1}$$

The condition for equilibrium of any infinitely small element of the stringer $\Gamma_k = \{x = (2k+1)\ b,\ |y| < a\}$, perpendicular to the plate along its entire length, the absence of resistance of the stringer to bending, and the continuity of the displacement components and relative elongation $\varepsilon_y = \partial v/\partial y$ in crossing the stringer axis have the form

$$h\left(\tau_{xy}^+ - \tau_{xy}^-\right) - N'(y) = 0, \quad \sigma_x^+ - \sigma_x^- = 0; \tag{2}$$

$$u^+ + iv^+ = u^- + iv^-, \quad \varepsilon_y^+ = \varepsilon_y^- = \varepsilon_y^0. \tag{3}$$

Expression (2) together with equation $N(y) = E_0 S_0 \varepsilon_y^0 = E_0 S_0 \left(\frac{\partial v}{\partial y}\right)^+$ lead to the relation

$$h \int_{-a}^{y} \left[(\sigma_x + i\tau_{xy})^+ - (\sigma_x + i\tau_{xy})^-\right] dy - iE_0 S_0 \left(\frac{\partial v}{\partial y}\right)^+ = 0. \tag{4}$$

Equalities (3) and (4) determine the boundary conditions for the coutours Γ_k. Here, they are the same as in [1].

Let us introduce the Kolosov—Muskhelishvili functions $\varphi(z)$ and $\psi(z)$. According to [6]

$$\begin{gathered} \sigma_x + \sigma_y = 2[\Phi(z) + \overline{\Phi(z)}],\ \sigma_y - \sigma_x + 2i\tau_{xy} = 2[\bar{z}\Phi'(z) + \Psi(z)], \\ 2G(u + iv) = \varkappa\varphi(z) - z\overline{\varphi'(z)} - \overline{\psi(z)},\ \Phi(z) = \varphi'(z),\ \Psi(z) = \psi'(z), \\ z = x + iy,\ 2G = E/(1+\nu),\ \varkappa = (3-\nu)/(1+\nu). \end{gathered} \tag{5}$$

Then, relations (3) and (4) together with the condition for the absence of stress on the contours of the openings $\gamma_k = \{2kb + \rho e^{i\theta},\ 0 \leqslant \theta \leqslant 2\pi\}$ are transformed to the following boundary-value problem:

$$\begin{gathered} H_1^+(t) - H_1^-(t) = 0, \\ (\varkappa + 1)[\varphi^+(t) - \varphi^-(t)] + \lambda_0 \operatorname{Re} H_2^+(t) = 0, \\ t \in \Gamma_k, \\ \Phi(s) + \overline{\Phi(s)} - e^{2i\theta}[\bar{s}\Phi'(s) + \Psi(s)] = 0,\ s \in \gamma_k, \end{gathered} \tag{6}$$

where $H_1(z) = \varkappa\varphi(z) - z\overline{\varphi'(z)} - \overline{\psi(z)};$

$$H_2(z) = \varkappa\Phi(z) - \overline{\Phi(z)} + z\overline{\Phi'(z)} + \overline{\Psi(z)},\ \lambda_0 = E_0 S_0/2Gh.$$

We assume that

$$\begin{gathered} \varphi(z) = \sum_{k=-\infty}^{\infty} \varphi_{1,k}(z) + \varphi_2(z) + \frac{p}{4} z, \\ \psi(z) = \sum_{k=-\infty}^{\infty} \{\varphi_{3,k}(z) + [z - (4k+2)b]\,\varphi_{1,k}'(z)\} + \psi_2(z) + \frac{p}{2} z, \\ \varphi_{jk}(z) = \frac{1}{2\pi i} \int_{b_k} f_j(\xi)(\xi - z)^{-1}\, d\xi, \quad j = 1, 3, \\ b_k = \{(2k+1)b - ia,\ (2k+1)b + ia\}. \end{gathered} \tag{7}$$

The functions φ_{jk} are discontinuous when crossing the stringer line. The first relation (6) turns out to be satisfied, if $f_3(\xi) = \varkappa\overline{f_1(\xi)}$, and the second is transformed into the form

$$\begin{gathered} f_1(t) + \frac{\lambda_0}{1+\varkappa} \operatorname{Re}\left\{\varkappa\Phi_2(t) - \overline{\Phi_2(t)} + \overline{t\Phi_2'(t)} + \overline{\Psi_2(t)}\right. \\ \left. + \frac{1}{2\pi i} \int_{-ai}^{ai} \left[\varkappa f_1'(\xi) + \overline{f_1'(\xi)}\left(1 - (\xi - t)\frac{d}{dt}\right)\right] L_1(\xi - t)\, d\xi\right\} + \frac{\lambda_0 p}{4} = 0, \end{gathered} \tag{8}$$

where $L_1(\xi) = (\pi/2b)\cot(\pi\xi/2b)$.

Evidently, $\operatorname{Im} f_1(t) = 0$.

For periodic functions $\Phi_2(z)$ and $\Psi_2(z)$ and functions of the density $f_1(\xi)$ with integral representation (7), we adopt the expression [7]

$$\Phi_2(z) = \rho \sum_{m=0}^{\infty} \varepsilon^{2m} \left[A_m'(z) + \frac{1}{\pi i} \sum_{k=0}^{\infty} (2k+1)\alpha_{k+1}(2b)^{-2k-2} \int_{\gamma_0} A_m(\xi)(\xi - z)^{2k} d\xi \right], \tag{9}$$

$$\Psi_2(z) = \rho \sum_{m=0}^{\infty} \varepsilon^{2m} \left[C_m'(z) + \frac{1}{\pi i} \sum_{k=0}^{\infty} (2k+1)\alpha_{k+1}(2b)^{-2k-2} \times \int_{\gamma_0} C_m(\xi)(\xi - z)^{2k} d\xi \right] - \Phi_2(z) - z\Phi_2'(z),$$

$$f_1(\xi) = a \sum_{m=0}^{\infty} \varepsilon^{2m} g_m(\xi),\ \varepsilon = \rho/2b,\ \alpha_k = \sum_{n=0}^{\infty} n^{-2k}.$$

The functions $A_m(\xi)$ and $C_m(\xi)$ are assumed to be analytic outside γ_0 and vanishing at infinity.

Substituting (9) into (8) and the third relation (6) and equating the coefficients with identical powers of ε, we obtain an infinite system of equations of the form (in what follows, $t = at_1$, $\xi = a\xi_1$, $s = \rho s_1$, and the index 1 is dropped)

$$g_m(t) - \frac{\lambda\varkappa}{\pi} \int_{-1}^{1} \frac{g_m'(\xi)}{\xi - t} d\xi + \frac{\lambda}{\pi} \int_{-1}^{1} g_m(\xi) Q(\xi - t)\, d\xi + \lambda \operatorname{Re}\left[\varkappa A_m'(t) - 2\overline{A_m'(t)} + 2it\overline{A_m''(t)} + \overline{C_m'(t)}\right] + F_{1,m}(t) = 0, \tag{10}$$

$$C_m'(s) + (1 + s^2) A_m'(s) - s^2 A_m'(s) + (s - s^{-1})\overline{A_m''(s)} + (\varkappa + 1 - s^2)\overline{U_m(s)} - s^2 U_m(s) - \overline{V_m(s)} + F_{2,m}(s) = 0,$$

$$|t| < 1,\ s = e^{i\theta},\ m = 0, 1, 2, \ldots,$$

where

$$F_{1,0}(t) = \frac{\lambda p}{4}(\varkappa + 1);\quad F_{2,0}(s) = \frac{p}{2}(1 - s^2);$$

$$F_{1,m}(t) = \sum_{k=0}^{m-1} (2k+1)\alpha_{k+1}\left[\varkappa G_{l,2k}(t) - 2\overline{G_{l,2k}(t)} - 4itk\overline{G_{l,2k-1}(t)} + H_{l,2k}(t)\right];$$

$$F_{2,m}(s) = -\sum_{k=0}^{\infty} (2k+1)\alpha_{k+1}\left[(1 + s^2)\overline{G_{l,2k}(s)} + s^2 G_{l,2k}(s) + 2k(s - s^{-1})\overline{G_{l,2k-1}(s)} - \overline{H_{l,2k}(s)}\right] \quad (l = 2m - 2k - 2);$$

$$G_{mj}(z) = \frac{1}{\pi i} \int_{\gamma_0} A_m(\xi)(\xi - z)^j d\xi,\quad H_{mj}(z) = \frac{1}{\pi i} \int_{\gamma_0} C_m(\xi)(\xi - z)^j d\xi;$$

$$U_m(s) = \frac{1}{\pi i} \int_{-1}^{1} f_m'(\tau) L_2(\tau, s)\, d\tau;$$

$$V_m(s) = \frac{1}{\pi i} \int_{-1}^{1} f_m'(\tau) \left[1 - i(\tau - \sin\theta)\frac{d}{ds}\right] L_2(\tau, s)\, d\tau;$$

$$L_2(\tau, s) = \frac{\pi a}{2b} \operatorname{tg} \frac{\pi}{2b}(\rho s - ia\tau);\quad \lambda = \lambda_0 [a(\varkappa + 1)]^{-1};$$

$$Q(\xi) = -\sum_{k=1}^{\infty} \beta_k \xi^{2k-1};\quad \beta_k = (\varkappa - 2k)\frac{2^{2k}}{(2k)!}\left(\frac{\pi a}{2b}\right)^{2k} B_{2k};$$

B_{2k} are the Bernoulli numbers [8].

In order to solve Eqs. (10), we will use Muskhelishvili's method [6, 7], proposed for studying a plane, containing a circular opening, and the Mul'toop—Kalandiya method [1] for numerically integrating singular integrodifferential equations. We seek the functions $A_m(s)$ and $C_m(s)$ in the form of the series

$$A_m(s) = -p \sum_{n=1}^{\infty} \frac{a_{mn}}{2n-1} s^{-2n+1},\quad C_m(s) = -p \sum_{n=1}^{\infty} \frac{c_{mn}}{2n-1} s^{-2n+1}$$

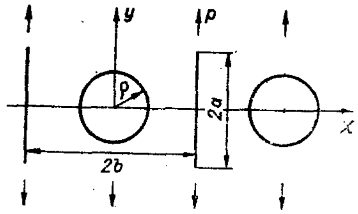

Fig. 1

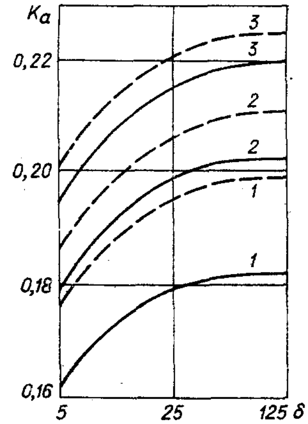

Fig. 2

TABLE 1

	e_2 / e_1 / δ	0,2			0,3		
		0,8	1	1,2	0,8	1	1,2
K_1 θ=0	0	3,0158	3,0065	3,0031	3,0801	3,0328	3,0158
	5	3,0059	2,9976	2,9945	3,0484	3,0066	2,0918
	25	3,0053	2,9964	2,9932	3,0438	3,0027	2,9884
	125	3,0050	2,9961	2,9929	3,0427	3,0019	2,9875
K_2 $\theta=\frac{\pi}{2}$	0	0,8398	0,8870	0,9176	0,7675	0,7982	0,8399
	5	0,8395	0,8864	0,9168	0,7688	0,7979	0,8384
	25	0,8395	0,8862	0,9167	0,7690	0,7979	0,8383
	125	0,8395	0,8862	0,9167	0,7690	0,7979	0,8382

and the function $g_m(\xi)$ in the form of an interpolating trigonometric polynomial

$$g_m(\cos\theta)=\frac{p}{N+1}\sum_{n=1}^{N} g_{mn}\frac{(-1)^{n+1}\cos\theta_n\cdot\sin(N+1)\theta}{\cos\theta-\cos\theta_n},\quad \theta_n=\pi n/(N+1).$$

The problem is reduced to an infinite algebraic system for the coefficients of the expansions introduced above, which is not written out here.

Let us estimate the stresses on the contours of the openings and the effect of the openings on the coefficient of stress intensity at the ends of stringers. According to (5), (7), (9), for $z = \rho e^{i\theta}$, we have

$$\sigma_\theta=4\,\mathrm{Re}\,\Phi(s)=p+4p\,\mathrm{Re}\sum_{m=0}^{\infty}\varepsilon^{2m}\left\{\sum_{n=1}^{\infty}a_{mn}s^{-2n}+2\varepsilon^2\sum_{k=0}^{\infty}\alpha_{k+1}\sum_{n=1}^{k+1}a_{m-k,n}\right.$$
$$\left.\times C_{2k+1}^{2n-1}s^{2k-2n+2}-\frac{1}{2\pi}\int_{-1}^{1}g'_m(\xi)\,L_2(\xi,s)\,d\xi\right\},\quad C_k^n=k![n!(k-n)!]^{-1}.$$

The tangential stress $\tau_{xy}=\mathrm{Im}[\bar z\Phi'(z)+\Psi(z)]$ in traversing the stringer axis has a discontinuity, which, according to Sokhotskii's equations, turns out to equal $\tau_{xy}^{+}-\tau_{xy}^{-}=-(\varkappa+1)\,f_1'(y)$, as a result of which just as in [3], we obtain the following expression for the coefficient of intensity of stress at the end of a stringer

$$K_{st}=\lim_{y\to a}\sqrt{a-y}\,\tau_{xy}^{+}(y)=-\frac{\varkappa+1}{2}\sqrt{a}\lim_{y\to 1}\sqrt{1-y}\,f_1'(y)=\frac{\varkappa+1}{4}\sqrt{2a}\sum_{m=0}^{\infty}\varepsilon^{2m}\lim_{y\to 1}\frac{g_m(y)}{\sqrt{1-y^2}}$$

The effect of stringers on the stress distribution is illustrated by the results of calculations carried out retaining in the expansions (9) terms having indices m = 0, 1, 2. Table 1 presents the values of the coefficients of stress concentration $K_1 = \sigma_\theta/p|_{\theta=0}$ and $K_2 = \sigma_\theta/p|_{\theta=\pi/2}$ with the parameters $e_1 = a/\rho$, $e_2 = a/b$, $\delta = 2E_0S_0/Eah$ varying. For $\delta = 0$, we obtain the results in [7]. The role of stringers in removing stresses on contours of openings becomes all the more significant, the closer the openings are situated to one another and, in addition, with increasing length and rigidity of stringers, their action on the change in the coefficient K_1 is greater than on the change in K_2. For example, for the case $e_1 = 1, 2$, when the rigidity δ increases, the coefficient K_1 with $e_2 = 0.3$ decreases by a factor of three more rapidly than for $e_2 = 0.2$. Moreover, for values of e_1 less than one, in the case $e_2 = 0.3$, when δ is increased and K_1 decreases correspondingly, some increase is observed in K_2.

The behavior of the coefficient of stress intensity at the end of a stringer is illustrated in Fig. 2, where $K_\alpha = K_{st}[p(\varkappa + 1)]^{-1}$. The quantity e_1 assumes the values 0.8, 1, 1.2, and e_2 assumes the values 0.2 (continuous lines 1-3, respectively) and 0.3 (dashed lines). The increase in the length and rigidity of a stringer, evidently, removes the stress on the contour of the opening, but greatly increases the stress intensity at its ends.

We honor the memory of L. M. Kupshin, to whom we are indebted for many suggestions and constant attention to our work.

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